Joint Beamforming And Power Splitting For Multi-User MISO SWIPT System

Qingjiang Shi[1], Weiqiang Xu[1], Dapeng Li[2]
[1]School of Info. Sci. & Tech., Zhejiang Sci-Tech University, Hangzhou 310018, China
[2]Nanjing University of Posts and Telecommunications, Nanjing 210003, China

***Abstract*—This paper considers a multi-user MISO broadcast system for simultaneous wireless information and power transfer (SWIPT), where multiple receivers receive energy and information simultaneously from the signal broadcast by a base station based on dynamic power splitting (DPS). We aim to minimize the total transmission power by joint beamforming and power splitting under both the signal-to-interference-plus-noise ratio (SINR) constraints and energy harvesting (EH) constraints. The joint beamforming and power splitting problem is a nonconvex problem which is complicated by the coupling of transmit beamformers and power splitting ratios. We apply semidefinite relaxation method to the problem and derive a relaxed but convex problem. By dealing with the dual problem of the relaxed problem, we propose a joint beamforming and power splitting algorithm and moreover present some conditions under which the relaxation solution is optimal. Simulation results show that more transmission power can be saved by using dynamic power splitting as compared with using fixed power splitting.**

***Index Terms*—Simultaneous wireless information and power transfer, energy harvesting, beamforming, power splitting, semidefinite relaxation.**

I. Introduction

Green radio has been an important research topic under the pressure of increasingly severe energy crisis and environmental pollution. Besides energy saving technologies, utilization of environmental energy sources is also a promising way to realize green radio. Different from other environmental energy sources such as wind, solar, geothermal heat, etc., radio signal is not only a potential green energy source but also can carry wireless information. Recently, simultaneous wireless information and power transfer (SWIPT) has gained a great deal of attentions [1]–[10] and brings new challenges on design of transmission schemes and protocols that utilizes the harvested energy efficiently.

Initial research works in the field of SWIPT focus on point-to-point single-antenna SWIPT systems. In [1], [2], the tradeoffs between information rate and power transfer are characterized for point-to-point single–antenna SWIPT system under different channel setups. [3] considers a more complex point-to-point single-antenna SWIPT system subject to time varying co-channel interference, where the optimal receiving mode (either information decoding or energy harvesting) switching rule is derived to achieve various trade-offs between wireless information transfer and energy harvesting. Multi-antenna SWIPT system has been investigated in [4]–[6]. [4] considers a MIMO broadcast channel with two separated or co-located receivers (one is information receiver and the other is energy receiver), where the rate-energy region is characterized and the optimal transmission schemes are investigated. [5] studies quality of service (QoS)-constrained robust beamforming for a two-user MISO SWIPT system with separated information/energy receivers. [6] extends [4], [5] and investigates the optimal beamforming strategy for multi-user MISO SWIPT system with *separated* information/energy receiver. [7], [8] considers SWIPT over multiple access channel and relay channel. [9] proposes a dynamic power splitting (DPS) scheme, where the received signal is split with adjustable power ratio for energy harvesting and information decoding, to mitigate the practical limitation of hardware realization of co-located information/energy receiver.

In this paper, we consider a DPS-based multi-user MISO SWIPT system, where a base station with multiple antennas transmits signals to multiple single antenna receivers each of which can receive both information and energy based on dynamic power splitting. Hence, our work is different from the most related work [6] where the receivers are scheduled to be either an information receiver or an energy receiver. We seek to minimize the transmission power by joint beamforming and power splitting subject to two types of QoS constraints: one is the SINR constraint while the other is the energy harvesting constraint. We formulate this as a nonconvex problem which is complex due to the coupling of the beamformers and power splitting ratios in the QoS constraints. We apply the popular semidefinite relaxation (SDR) technique [10] to the nonconvex problem and solve the relaxed problem by dealing with its dual problem. We show some mild conditions under which the relaxation is tight, i.e., the SDR solution is also the optimal solution to the original nonconvex problem. Simulation results show that more transmission power can be saved by using dynamic power splitting as compared with using fixed power splitting.

The remainder of this paper is organized as follows. The system model and problem formulation are presented in Section II. In Section III, we propose the semidefinite relaxation based solution and perform an analysis on the tightness of the relaxation. Section IV presents some simulation results, while Section V concludes the paper.

Notations: In this paper, scalars are denoted by lower-case letters, bold-face lower-case letters are used for vectors, and bold-face upper-case letters for matrices. For a square matrix $\mathbf{A}$, $\mathrm{Tr}(\mathbf{A})$ and $\mathbf{A}^H$ denote its trace and conjugate transpose respectively, while $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is a positive

semidefinite matrix. $\mathbf{I}_n$ denotes an n by n identity matrix. $||\boldsymbol{x}||$ is the Euclidean norm of a complex vector $\boldsymbol{x}$. The distribution of a circularly symmetric complex Gaussian (CSCG) random vector variable with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}$ is denoted by $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{C})$, and "$\sim$" stands for "distributed as". $\mathbb{C}^{m\times n}$ denotes the space of $m \times n$ complex matrices.

II. System Model and Problem formulation

Consider a multi-user MISO downlink SWIPT system of K users, where the base station (BS) is equipped with $N_t > 1$ antennas and each user is equipped with a single antenna. We assume the BS transmits a combination signal of all users' using a set of beamforming vectors $\boldsymbol{v}_k \in \mathbb{C}^{N_t\times 1}$. Each user k receives signal and splits it into two parts. One part is used for further signal processing and symbol detection, and the other part is driven to the energy harvesting circuit for conversion to DC voltage and energy storage. The system model is depicted in Fig. 1 where ρ_k is the power splitting ratio which means a fraction, ρ_k, of the received signal energy is used for information decoding while the remaining energy is for energy harvesting.

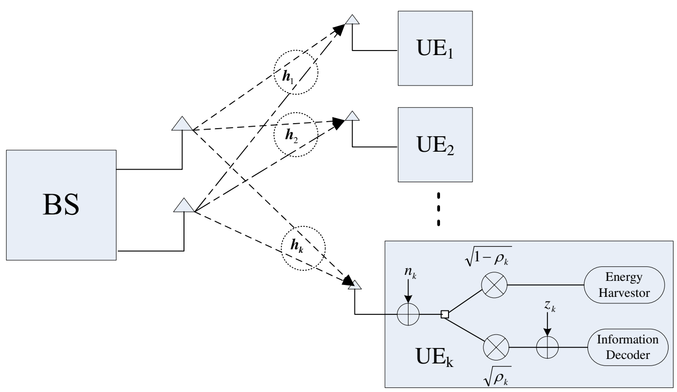

Fig. 1. A power splliting-based MISO SWIPT system. UE k splits the received energy with a power spliting ratio ρ_k into two parts for energy information decoding and energy harvesting.

Assume frequency-nonselective Raleigh block fading channels between the BS and users. Let $s_k \sim \mathcal{CN}(0,1)$ be the transmitted symbol intended for user k. Then the received signal at user k before power splitting can be mathematically expressed as

$$y_k = \boldsymbol{h}_k^H \sum_{j=1}^{K} \boldsymbol{v}_j s_j + n_k \tag{1}$$

where $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is an additional white Gaussian noise (AWGN) introduced by the receiving antenna, $\boldsymbol{h}_k$ denotes the channel between the BS and user k. After power spliting, the signal for information decoding at user k is modeled as

$$y_k^{ID} = \sqrt{\rho_k}\left(\boldsymbol{h}_k^H \sum_{j=1}^{K} \boldsymbol{v}_j s_j + n_k\right) + z_k \tag{2}$$

where $z_k \sim \mathcal{CN}(0, \delta_k^2)$ is an AWGN due to RF to baseband signal conversion [9], and the signal for energy harvesting is

$$y_k^{EH} = \sqrt{1-\rho_k}\left(\boldsymbol{h}_k^H \sum_{j=1}^{K} \boldsymbol{v}_j s_j + n_k\right). \tag{3}$$

We assume all the noises n_k's, z_k's and symbols s_k's are independent from each other. Then the SINR metric for symbol detection at user k is given by:

$$\mathrm{SINR}_k \triangleq \frac{\rho_k |\boldsymbol{h}_k^H \boldsymbol{v}_k|^2}{\rho_k \sum_{j\neq k} |\boldsymbol{h}_k^H \boldsymbol{v}_j|^2 + \rho_k \sigma_k^2 + \delta_k^2}, \tag{4}$$

and the harvested power at user k is

$$E_k = \zeta_k (1-\rho_k)\Big(\sum_{j=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{v}_j|^2 + \sigma_k^2\Big). \tag{5}$$

where $\zeta_k \in (0\ 1]$ denotes the conversion efficiency of the energy harvesting unit.

Different from a general wireless communication system, there are two types of QoS requirement in an SWIPT system. To guarantee successful symbol detection, it is required for each user k that the achieved SINR should be higher than a threshold γ_k, corresponding to SINR constraint. Also, to ensure substantiable power supply for user k, the harvested power E_k should be higher than a threshold $\hat{e}_k$, corresponding to EH constraint. Under these two kinds of QoS constraints, we seek to minimize the transmission power by joint beamforming and power splitting. The corresponding optimization problem, called QoS constrained power minimization (QCPM) problem, can be mathematically written as

$$\begin{aligned} \min_{\{\boldsymbol{v}_k, \rho_k\}} \ & \sum_{k=1}^{K} ||\boldsymbol{v}_k||^2 \\ \text{s.t.} \ & \frac{\rho_k |\boldsymbol{h}_k^H \boldsymbol{v}_k|^2}{\sum_{j\neq k} \rho_k |\boldsymbol{h}_k^H \boldsymbol{v}_j|^2 + \rho_k \sigma_k^2 + \delta_k^2} \geq \gamma_k, \\ & (1-\rho_k)\Big(\sum_{j=1}^{K} |\boldsymbol{h}_k^H \boldsymbol{v}_j|^2 + \sigma_k^2\Big) \geq e_k^2, \\ & 0 \leq \rho_k \leq 1, k \in \mathcal{K}. \end{aligned} \tag{6}$$

where $e_k^2 \triangleq \frac{\hat{e}_k}{\zeta_k}$ and $\mathcal{K} \triangleq \{1, 2, \ldots, K\}$.

We assume problem (6) is feasible throughout this paper and moreover $\gamma_k \geq 1$ and $e_k^2 \geq \sigma_k^2$ for all k. Clearly, problem (6) is nonconvex and difficult to solve. Particularly, even when ρ_k's are fixed, the problem is still more difficult than the general SINR-constrained power minimization problem (which can be cast as a second order cone programming (SOCP) by phase rotation and thus solved efficiently [11]) due to the extra nonconvex EH constraints.

III. Semidefinite Relaxation Based Solution

In this section, we propose a semidefinite relaxation based solution to problem (6). Specifically, we first relax the non-convex problem (6) as a convex problem by using semidefinite relaxation technique. Then we solve the resultant problem in

its dual domain and give conditions under which the relaxation is tight.

A. Semidefinite Relaxation and Dual Problem

Define $\mathbf{X}_k = \boldsymbol{v}_k \boldsymbol{v}_k^H$. Then problem (6) can be equivalently transformed to

$$\begin{aligned} &\min_{\{\mathbf{X}_k,\rho_k\}} \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{X}_k) \\ &\text{s.t. } \frac{1}{\gamma_k}\boldsymbol{h}_k^H \mathbf{X}_k \boldsymbol{h}_k - \sum_{j\neq k} \boldsymbol{h}_k^H \mathbf{X}_j \boldsymbol{h}_k \geq \sigma_k^2 + \frac{\delta_k^2}{\rho_k}, \\ &\quad \sum_{j=1}^{K} \boldsymbol{h}_k^H \mathbf{X}_j \boldsymbol{h}_k \geq \frac{e_k^2}{1-\rho_k} - \sigma_k^2, \\ &\quad \mathbf{X}_k \succeq 0, \\ &\quad \mathrm{Rank}(\mathbf{X}_k) = 1, \\ &\quad 0 \leq \rho_k \leq 1, k \in \mathcal{K}. \end{aligned} \tag{7}$$

Problem (7) is still nonconvex due to the rank constraint. By relaxing the rank constraint (i.e., dropping it), we obtain the following optimization problem

$$\begin{aligned} &\min_{\{\mathbf{X}_k,\rho_k\}} \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{X}_k) \\ &\text{s.t. } \frac{1}{\gamma_k}\boldsymbol{h}_k^H \mathbf{X}_k \boldsymbol{h}_k - \sum_{j\neq k} \boldsymbol{h}_k^H \mathbf{X}_j \boldsymbol{h}_k \geq \sigma_k^2 + \frac{\delta_k^2}{\rho_k}, \\ &\quad \sum_{j=1}^{K} \boldsymbol{h}_k^H \mathbf{X}_j \boldsymbol{h}_k \geq \frac{e_k^2}{1-\rho_k} - \sigma_k^2, \\ &\quad \mathbf{X}_k \succeq 0, \\ &\quad 0 \leq \rho_k \leq 1, k \in \mathcal{K}. \end{aligned} \tag{8}$$

Note that $\frac{1}{\rho_k}$ and $\frac{1}{1-\rho_k}$ are convex functions of ρ_k when $0 \leq \rho_k \leq 1$. Hence, the constraint set is convex and thus problem (8) is a convex problem. Although problem (8) can be cast as a *linear cone programming* and thus solved by some off-the-shelf optimization tools, to facilitate analysis and reduce the problem size, we instead consider its dual problem which is simpler.

First we present the following two lemmas but omit the proofs here.

Lemma 1: Strong duality holds for problem (8).

Lemma 2: Let $x, y > 0$. The problem

$$\min_{0\leq\rho\leq 1} \frac{x^2}{\rho} + \frac{y^2}{1-\rho}$$

has a closed form solution $\rho^* = \frac{\boldsymbol{x}}{\boldsymbol{x}+\boldsymbol{y}}$ and its optimal value is $(x+y)^2$.

Let λ_k and μ_k be respectively the dual variable associated with the first and second constraint of problem (8). Let $\mathbf{X}$ denote the set of $\{\mathbf{X}_k\}$, $\boldsymbol{\rho}$ denote the set of $\{\rho_k\}$, and similarly for $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. Based on Lemma 1 and Lagrange dual theory [12], we have the dual problem of problem (8) as follows

$$\max_{\{\lambda_k,\mu_k\geq 0\}} \min_{\{\mathbf{X}_k\succeq 0, 0\leq\rho_k\leq 1\}} \mathbb{L}(\mathbf{X},\boldsymbol{\rho},\boldsymbol{\lambda},\boldsymbol{\mu})$$

where $\mathbb{L}(\mathbf{X},\boldsymbol{\rho},\boldsymbol{\lambda},\boldsymbol{\mu})$ shown in (9) is the Lagrange function. Further, by using Lemma 2 and noting that $\min_{\mathbf{Y}\succeq 0} \mathrm{Tr}(\mathbf{AY})$ is zero when $\mathbf{A} \succeq 0$ otherwise negative infinity, we can explicitly write out the dual problem and have the following proposition.

Proposition 1: The dual problem of problem (8) is

$$\begin{aligned} &\max_{\{\lambda_k,\mu_k\}} \sum_{k=1}^{K} (\sqrt{\lambda_k}\delta_k + \sqrt{\mu_k}e_k)^2 + (\lambda_k - \mu_k)\sigma_k^2 \\ &\text{s.t. } \mathbf{I}_{N_t} + \sum_{j\neq k} (\lambda_j - \mu_j)\boldsymbol{h}_j\boldsymbol{h}_j^H - \left(\frac{\lambda_k}{\gamma_k} + \mu_k\right)\boldsymbol{h}_k\boldsymbol{h}_k^H \succeq 0, \\ &\quad \lambda_k, \mu_k \geq 0, k \in \mathcal{K}. \end{aligned} \tag{10}$$

And the optimal primal variable ρ_k^*, dual variables λ_k^*, μ_k^*, $k \in \mathcal{K}$ are related by

$$\rho_k^* = \frac{\sqrt{\lambda_k^*}\delta_k}{\sqrt{\lambda_k^*}\delta_k + \sqrt{\mu_k^*}e_k}, k \in \mathcal{K}.$$

Since ρ_k and $1-\rho_k$ must be positive, we have the following Corollary.

Corollary 1: The optimal dual variables are all positive, i.e., $\lambda_k^*, \mu_k^* > 0$, $k \in \mathcal{K}$.

B. When The Relaxation Is Tight?

Throughout the rest of the paper, we denote the optimal dual variables by λ_k^*'s and μ_k^*'s, and denote the optimal primal variables by ρ_k^*'s and $\mathbf{X}_k^*$'s. Define

$$\mathbf{Y}_k \triangleq \mathbf{I}_{N_t} + \sum_{j\neq k} (\lambda_j - \mu_j)\boldsymbol{h}_j\boldsymbol{h}_j^H - \left(\frac{\lambda_k}{\gamma_k} + \mu_k\right)\boldsymbol{h}_k\boldsymbol{h}_k^H, k \in \mathcal{K}.$$

and denote the optimal $\mathbf{Y}_k$ by $\mathbf{Y}_k^*$ (corresponding to the optimal dual variables).

In terms of the complementarity condition (one of first-order optimality necessary condition [12]), we have $\mathbf{Y}_k^*\mathbf{X}_k^* = 0$ for all k, that is, the columns of $\mathbf{X}_k^*$ lie in the null space of $\mathbf{Y}_k^*$. Hence, when $\mathrm{Rank}(\mathbf{Y}_k^*) = N_t - 1$, we always have $\mathrm{Rank}(\mathbf{X}_k^*) = 1$ and factorizing $\mathbf{X}_k^*$ as $\mathbf{X}_k^* = \boldsymbol{v}_k^*\left(\boldsymbol{v}_k^*\right)^H$ yields the optimal solution of problem (6) (i.e., $\boldsymbol{v}_k^*$), implying that the semidefinite relaxation is tight. In the following, we present some cases when $\mathrm{Rank}(\mathbf{Y}_k^*) = N_t - 1$. First, an important property of $\mathbf{Y}_k^*$'s is stated in the following lemma. The proof is simple and omitted here.

Lemma 3: $\mathbf{Y}_k^*$'s are positive semidefinite but not positive definite.

Lemma 3 implies the following proposition.

Proposition 2: If $N_t = 2$, i.e., the BS is equipped with two antennas, $\mathrm{Rank}(\mathbf{Y}_k^*) = N_t - 1$, $\forall k \in \mathcal{K}$.

By noting that $\mathbf{Y}_k^*$ is in the form of a positive definite matrix subtracting a rank one positive semidefinite matrix when $\lambda_k^* \geq \mu_k^*$ holds for all k, we have Proposition 3.

Proposition 3: If $\lambda_k^* \geq \mu_k^*$ for all k, then $\mathrm{Rank}(\mathbf{Y}_k^*) = N_t - 1$, $k \in \mathcal{K}$.

The following proposition gives a condition under which it must hold that $\lambda_k^* \geq \mu_k^*$.

$$\mathbb{L}(\mathbf{X},\boldsymbol{\rho},\boldsymbol{\lambda},\boldsymbol{\mu}) \triangleq \sum_{k=1}^{K} \mathrm{Tr}\left(\left(\mathbf{I}_{N_t} + \sum_{j\neq k}(\lambda_j-\mu_j)\boldsymbol{h}_j\boldsymbol{h}_j^H - \left(\frac{\lambda_k}{\gamma_k}+\mu_k\right)\boldsymbol{h}_k\boldsymbol{h}_k^H\right)\mathbf{X}_k\right) + \sum_{k=1}^{K}\left(\lambda_k\left(\frac{\delta_k^2}{\rho_k}+\sigma_k^2\right)+\mu_k\left(\frac{e_k^2}{1-\rho_k}-\sigma_k^2\right)\right) \tag{9}$$

Proposition 4: If γ_k, δ_k, σ_k, e_k, $k\in\mathcal{K}$ satisfy

$$2\delta_k e_k(\gamma_k-1)+\gamma_k(\delta_k^2+\sigma_k^2)-(e_k^2-\sigma_k^2)\geq 0, k\in\mathcal{K}, \tag{11}$$

then $\lambda_k^*\geq\mu_k^*, k\in\mathcal{K}$.

Proof: A sketch of the proof is provided here. Assume for contrary there exists one k for which $\lambda_k^*<\mu_k^*$. Consider increasing λ_k^* by $\gamma_k x$ and decreasing μ_k^* by x where $x>0$, while fixing all other λ_j^*'s and μ_j^*'s, i.e., define $\lambda_k=\lambda_k^*+\gamma_k x$, $\lambda_j=\lambda_j^*$, $j\neq k$, and $\mu_k=\mu_k^*-x$, $\mu_j=\mu_j^*$, $j\neq k$. It can be easily verified that, λ_k's and μ_k's satisfy the matrix inequality constraints of the dual problem. Let p denote the objective function value of the dual problem at λ_k's and μ_k's, and p^* denote the objective function value of the dual problem at λ_k^*'s and μ_k^*'s. It can be easily checked that

$$\begin{aligned} p-p^* &= x\gamma_k(\delta_k^2+\sigma_k^2)-x(e_k^2-\sigma_k^2) \\ &\quad + 2\delta_k e_k\sqrt{(\lambda_k^*+\gamma_k x)(\mu_k^*-x)}-2\delta_k e_k\sqrt{\lambda_k^*\mu_k^*} \end{aligned} \tag{12}$$

Define the right-hand-side of eq. (12) as function $\phi(x)$. We have

$$\frac{d\phi(x)}{dx}=\gamma_k(\delta_k^2+\sigma_k^2)-(e_k^2-\sigma_k^2)+2\delta_k e_k\frac{\gamma_k\mu_k^*-\lambda_k^*-2\gamma_k x}{\sqrt{(\lambda_k^*+\gamma_k x)(\mu_k^*-x)}}. \tag{13}$$

Now we can verify that under condition (11) there must exist an arbitrarily small $x>0$ at which $\frac{d\phi(x)}{dx}>0$, implying $p>p^*$ at x and thus the proof is completed. ■

Remark 1: In terms of Proposition 2, 3, and 4, it is known that, if the condition (11) is satisfied or the BS is equipped with two antennas, then the semidefinite relaxation is tight.

C. Linear Cone Programming Based Beamforming Algorithm

The dual problem can be equivalently written as

$$\begin{aligned} &\max_{\{\lambda_k,\mu_k,w_k\}} \sum_{k=1}^{K}\lambda_k(\delta_k^2+\sigma_k^2)+\mu_k(e_k^2-\sigma_k^2)+2\delta_k e_k w_k \\ &\text{s.t. } \mathbf{I}_{N_t}+\sum_{j\neq k}(\lambda_j-\mu_j)\boldsymbol{h}_j\boldsymbol{h}_j^H-\left(\frac{\lambda_k}{\gamma_k}+\mu_k\right)\boldsymbol{h}_k\boldsymbol{h}_k^H\succeq 0, \\ &\quad \lambda_k\mu_k\geq w_k^2, k\in\mathcal{K}. \end{aligned} \tag{14}$$

The equivalence is due to the fact that the optimal λ_k's, μ_ks, w_k's must be positive in order to maximize the objective function of problem (14). Since the constraint $\lambda_k\mu_k\geq w_k^2$ is equivalent to

$$\lambda_k+\mu_k\geq\sqrt{(\lambda_k-\mu_k)^2+w_k^2},$$

which is a second-order cone constraint, problem (14) can be further cast as the following linear cone programming

$$\begin{aligned} &\max_{\{\lambda_k,\mu_k,w_k\}} \sum_{k=1}^{K}\lambda_k(\delta_k^2+\sigma_k^2)+\mu_k(e_k^2-\sigma_k^2)+2\delta_k e_k w_k \\ &\text{s.t. } \mathbf{I}_{N_t}+\sum_{j\neq k}(\lambda_j-\mu_j)\boldsymbol{h}_j\boldsymbol{h}_j^H-\left(\frac{\lambda_k}{\gamma_k}+\mu_k\right)\boldsymbol{h}_k\boldsymbol{h}_k^H\succeq 0, \\ &\quad \lambda_k+\mu_k\geq\sqrt{(\lambda_k-\mu_k)^2+w_k^2}, k\in\mathcal{K}. \end{aligned} \tag{15}$$

After solving problem (15), if all the matrices[1] $\mathbf{Y}_k^*$'s are of rank N_t-1, we can recover the primal solution. First, we obtain the normalized beamformers by matrix decomposition in terms of the complementarity condition $\mathbf{Y}_k^*\mathbf{X}_k^*=0$. Then we find the optimal power allocation by solving a linear system constructed from the SINR constraints by noting that the SINR constraints must be satisfied with equality at the optimality. The proposed algorithm is summarized in TABEL I.

TABLE I
JOINT BEAMFORMING AND POWER SPLITTING ALGORITHM

- **Input**: $\boldsymbol{h}_k$, σ_k^2, δ_k^2, γ_k, e_k^2, $k\in\mathcal{K}$
- **Output**: the transmit beamformers $\boldsymbol{v}_k^*$'s and power splitting ratios ρ_k^*'s.

1 solve problem (15) and obtain λ_k^*, μ_k^*, and $\mathbf{Y}_k^*$, $k\in\mathcal{K}$

2 set $\rho_k^*=\frac{\sqrt{\lambda_k^*}\delta_k}{\sqrt{\lambda_k^*}\delta_k+\sqrt{\mu_k^*}e_k}$, $k\in\mathcal{K}$

3 find the eigenvector $\boldsymbol{v}_k$ of $\mathbf{Y}_k^*$ with zero eigenvalue, $k\in\mathcal{K}$

4 construct a K by K matrix $\mathbf{A}$ with its (i,j)-th entry being

$$\mathbf{A}_{ij}=\begin{cases}\frac{1}{\gamma_i}|\boldsymbol{h}_i\boldsymbol{v}_i|^2, & i=j\\ -|\boldsymbol{h}_i\boldsymbol{v}_j|^2, & i\neq j\end{cases}$$

and a vector $\boldsymbol{b}$ with its k-th entry being

$$b_k=\sigma_k^2+\frac{\delta_k^2}{\rho_k^*}$$

.

5 find the optimal power allocation $\boldsymbol{p}$ by solving the linear system $\mathbf{A}\boldsymbol{p}=\boldsymbol{b}$

6 set $\boldsymbol{v}_k^*=\sqrt{p_k}\boldsymbol{v}_k$, $k\in\mathcal{K}$ where p_k is the k-th entry of $\boldsymbol{p}$

IV. Simulation Results

In this section, we numerically evaluate the performance of the proposed beamforming algorithm in terms of the minimum

[1]In our simulations, we always find that all $\mathbf{Y}_k^*$'s are of rank N_t-1, that is, the semidefinite relaxation is always tight.

total transmission power for different parameter configurations. The same parameters δ^2, σ^2, e^2, γ are set for different users, i.e., $\delta_k^2 = \delta^2$, $\sigma_k^2 = \sigma^2$, $e_k^2 = e^2$, $\gamma_k = \gamma$, $\forall k \in \mathcal{K}$. Moreover, we set $\sigma^2 = \delta^2 = -10$ dBm in all our simulations. All channel coefficients between transmit antennas and receive antennas are randomly and independently generated according to an identical CSCG distribution with zero mean and unit variance. In all our plots, each result is averaged over 100 channel realizations.

Figures 2 and 3 illustrate the average minimum transmission power required for different SINR and energy harvesting power requirement. Note that the condition (11) holds for all parameter combinations in this set of simulations. By checking the rank of $\mathbf{Y}_k^*$'s, we find that the relaxation is indeed tight. In addition, as expected, the required total transmission power increases with γ and e^2. Particularly, one can find from Fig. 3 that the total transmission power will converge as the EH power e^2 decreases. The reason is that for small e^2, the energy harvesting constraint will be inactive.

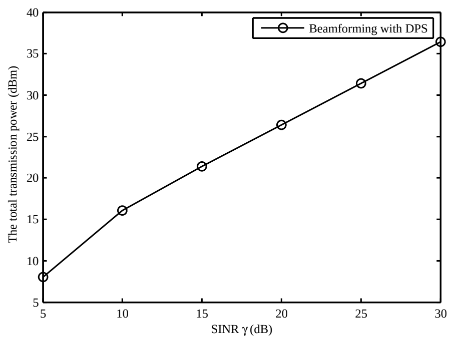

Fig. 2. The total transmission power increases with SINR threshold γ: $K = N_t = 4$, $e^2 = 0$ dBm.

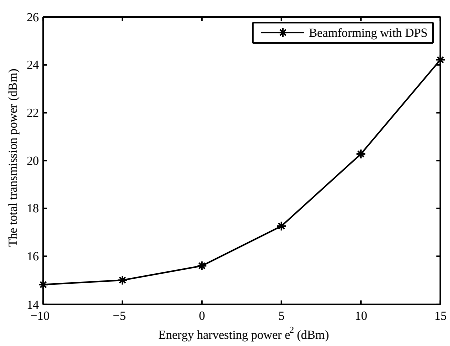

Fig. 3. The total transmission power increases with energy harvesting power: $K = N_t = 4$, $\gamma = 10$ dB.

Figures 4 illustrates the total transmission power required by beamforming with dynamic power splitting and beamforming with fixed power splitting ($\rho_1 = \rho_2 = 0.5$) when the BS is equipped with two antennas. From the plot, one can observe that using dynamic power splitting can save transmission power especially when the SINR requirement is high.

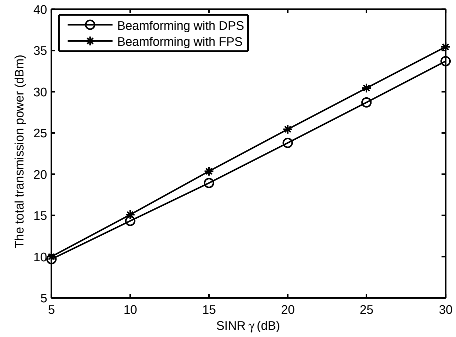

Fig. 4. Joint beamforming and power splitting versus beamforming with fixed power splitting: $K = N_t = 2$, $e^2 = 0$ dBm.

V. Conclusion

In this paper, we have considered QoS–constrained power minimization problem for multi-user MISO SWIPT system. A joint beamforming and power splitting algorithm has been proposed by applying semidefinite relaxation technique and Lagrange dual theory. We have shown some conditions under which the relaxation solution is optimal to the nonconvex joint beamforming and power splitting problem. From our simulations (not reported in this paper), we also note that the semidefinite relaxation is always tight even when the condition (11) is not satisfied. Hence, We conjecture that the semidefinite relaxation for this problem may be always tight. The proof for this is left as our future work.

References

[1] L. R. Varshney, "Transporting information and energy simultaneously," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, pp. 1612-1616, July 2008.

[2] P. Grover and A. Sahai, "Shannon meets Tesla: Wireless information and power transfer," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, pp. 2363-2367, June 2010.

[3] L. Liu, R. Zhang, and K. C. Chua, "Wireless information transfer with opportunistic energy harvesting," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 288-300, Jan. 2013.

[4] R. Zhang and C. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," Accepted in *IEEE Trans. Wireless Commun.*, 2013. Available online at [arXiv:1105.4999].

[5] Z. Xiang and M. Tao, Robust beamforming for wireless information and power transmission, *IEEE Wireless Commun. Letters*, vol. 1, no. 4, pp. 372-375, 2012.

[6] J. Xu, L. Liu, and R. Zhang, "Multiuser MISO beamforming for simultaneous wireless information and power transfer, accepted in *IEEE ICASSP*, 2013. Available online at [arXiv:1303.1911].

[7] B. K. Chalise, Y. D. Zhang, andM. G. Amin, Energy harvesting in an OSTBC based amplify-and-forward MIMO relay system, in *Proc. IEEE ICASSP*, pp. 3201-3204, Mar. 2012.

[8] A. M. Fouladgar and O. Simeone, "On the transfer of information and energy in multi-user systems," *IEEE Commun. Letters*, vol. 16, no. 11, pp. 1733-1736, Nov. 2012.

[9] X. Zhou, R. Zhang, and C. K. Ho, "Wireless Information and Power Transfer: Architecture Design and Rate-Energy Tradeoff," submitted for publication, 2012. Available online at [arXiv:1204.2035v2].

[10] Z.-Q. Luo, W.-K. Ma, A.M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Trans. Signal Process. Mag.*, vol. 27, no. 3, pp. 20-34, 2010.

[11] M. Bengtasson and B. Ottersten, "Optimal and suboptimal transmit beamforming," Chapter 18 in *Handbook of Antennas in Wireless Communications*, L. C. Godara, Ed., CRC Press, Aug. 2001.

[12] D. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA: Athena Scientific, 1999.